6.2 Current Transformers: Part 1.

Introduction:

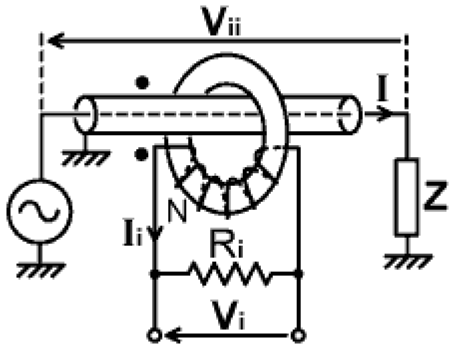
From a theoretical point of view, a current transformer is just a conventional transformer; but it holds a few surprises for the circuit designer because, for a given input current, it controls its own input voltage according to the secondary load. Since voltage and current have a reciprocal relationship, anyone used to thinking in terms of voltage may find that changes to a current transformer network sometimes have the opposite of the desired effect. Hence, given that there are peculiarities, we will spend some time looking at the basic properties of current transformers before attempting to incorporate them into bridges.

From a practical standpoint, it is usually best to design a current transformer so that there is very tight coupling between the primary and secondary windings; and in RF applications, this implies the use of a ferrite or powdered-iron toroidal core. A two-hole (binocular or 'pig-nose') core gives even tighter coupling, but the topology prevents single-layer winding of the secondary and is therefore usually rejected on the grounds that it gives rise to additional parasitic capacitance. The current transformer is effectively an impedance step-up device, where the impedance terminating the secondary appears in series with the signal path but scaled down according to the square of the turns ratio. It follows that if the secondary is not terminated, the transformer primary inductance will appear in series with the signal path (and the secondary voltage will be shifted in phase with respect to the primary current by approximately 90° as a result). Also, conversely, the transformer may effectively be removed from circuit by shorting the secondary, in which case the primary impedance falls practically to zero.

In toroidal transformers, a one turn winding is obtained by passing a wire once through the hole in the core. The primary of an RF current transformer is usually a single turn, made by pushing a short length of coaxial cable through the hole. The outer braid of the cable (when grounded) acts as a Faraday shield, preventing direct capacitive coupling between the primary and secondary windings. The shield must only be earthed at one end to avoid producing a shorted turn (earthing both ends of the Faraday shield is another way of switching-off the transformer). To reduce losses, the coaxial cable used should preferably have silver-plated or plain copper conductors and PTFE (Teflon) dielectric. Since the outer coating is also in the transformer field, this should be made from PTFE, FEP, ETFE or similar non-polar low-Tanδ material. If PVC covered cable is the only type available; the outer sheath can be removed, and the braid can be insulated by wrapping it with plumber's PTFE joint-sealing tape. Polyethylene dielectric is not recommended because it will melt during soldering and it may also melt in service if the transformer core gets hot. Faraday shielding is optional (and not always beneficial) and when it is used, it is not always shown explicitly on circuit diagrams.

Faraday shielded current transformer.

Notice, in the photograph below, that the Faraday shield is connected to the secondary network grounding point. The reason for doing so is that the chassis on which the transformer is mounted is also the return path for the primary current. The chassis has some inductance, and so a voltage that increases with frequency develops between the primary input and output ground connections. Since there is a capacitance of several pF between the Faraday shield and the transformer secondary winding, and the reactance of this capacitance falls with frequency; a disturbance of the transformer frequency response will occur unless the shield is maintained at the secondary network reference potential [for experimental confirmation see Amplitude response of conventional and maximally flat current transformers, by DWK].



In the diagram above, dots are used to indicate the phasing of the windings, i.e., the ends of the windings are marked in such a way that they can be considered to start (or pass into the hole) together. When this convention is observed, the arrow shown for the secondary voltage indicates the polarity in which it is in phase with the primary voltage (assuming ideal transformer behaviour). The primary voltage $\mathbf{V}_{ii}$ is, of course, the voltage drop that results from inserting the transformer into the signal path. The current in the secondary is 180° out of phase with the current in the primary, i.e. (insofar as the concept of 'flow' can be used), when current flows into one end of the primary, current flows out of the corresponding end of the secondary.

6.2-1 Ampere turns and transresistance:

In order to obtain design equations for the current transformer, it is sensible to begin by assuming that the reactance of the secondary winding is very large in comparison to the secondary load resistance and can therefore be ignored. This is a simplification that applies approximately when the transformer is working in the middle of its useful frequency range, for which reason we will sometimes refer to it as the *mid-band analysis*.

To obtain the mid-band input-output relationship, or at least a reasonable approximation to it, we start by assuming that the behaviour of a small toroidal transformer wound on core material chosen according to the manufacturer's instructions is ideal. The equations describing the transformer can then be deduced on the basis that the primary magneto-motive force (MMF) is equal to the secondary magneto-motive force; this quantity being simply the product of the current and the number of turns

(MMF is also known as "ampere-turns", which are its units). When dealing with RF current transformers; we can normally use a single number 'N' to express both the secondary turns and the turns ratio, because the number of primary turns is usually one. Thus, using the definitions given in the diagram above, we may write the ampere-turns rule:

$\mathbf{I} = N\,\mathbf{I}_i$

The secondary current thus being determined, we may obtain the secondary voltage:

$\mathbf{V}_i = \mathbf{I}_i\,R_i$

and hence:

$\mathbf{V}_i = \mathbf{I}\,R_i / N$

which gives the relationship between the primary current and the secondary voltage (neglecting losses and parasitic reactances). Notice that this expression tells us that the output voltage goes *down* as the number of secondary turns is increased, which is the opposite of what would be expected for a voltage transformer. As mentioned previously, this peculiar behaviour results because the secondary load controls the input impedance and hence the input voltage.

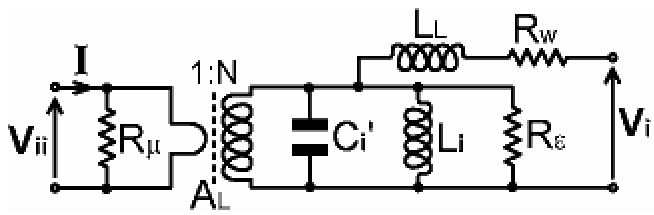
The quantity R_i/N is the transfer-resistance, or ***transresistance*** of the current-transformer network. It is analogous to the transresistance of an amplifier that gives an output in volts for an input in amps; i.e., it is a gain-figure with dimensions of Ohms. By analogy with the transconductance or 'mutual-conductance' G_m of an FET or valve, we will give it the symbol R_m. In the approximation that the transformer is ideal, it becomes the *nominal transresistance,* i.e.;

$R_{m(nom)} = R_i / N$

It is a useful figure to keep in mind when designing with current transformers. E.g.; A current transformer with a load resistance of 50Ω and a turns-ratio of 10, has a nominal transresistance of 5 Volts per Amp. A current transformer with a turns-ratio of 20 and a 56Ω load has $R_{m(nom)}$ = 2.8 V/A.

6.2-2. Detailed equivalent circuit:

Practical current transformers are, of course, far from ideal, and the issues that need to be taken into account when working with them can best be understood by considering the equivalent circuit shown below. This circuit is obtained by reversing the conventional model for a voltage transformer and referring all parameters except core loss to the secondary side. Core loss is best associated with the primary side because it still exists if the secondary winding is removed.



A_L = inductance factor

R_μ = core loss
L_i = coupled secondary inductance
L_L = leakage inductance
R_W = winding resistance
R_ε = dielectric loss
C_i' = "self-capacitance"

The equivalent circuit implies that the transformer leakage inductance (i.e., the inductance that manages to escape from inclusion in the coupling between windings) will appear in series with the output; so affecting the magnitude and phase of the voltage across the secondary load resistor (not shown). Our best course of action is to try to minimise it, which we have done to some extent by choosing to use a toroidal magnetic core; but it is also important to keep the primary winding close to the core (i.e., the hole in the core should be as small as possible), and to keep the magnetic-circuit path-length short (the outer core diameter should also be small). One consequence of leakage inductance is that the inductance obtained by measuring the secondary winding ($L_{sec}=L_i+L_L$) is always slightly greater than the inductance obtained by least-squares fitting to the transformer frequency-response data (the latter being a good approximation to L_i). The difference for a small toroidal core is usually in the region of 1 or 2%, being worst for low-permeability cores.

The winding resistance is also in series with the output, which would not be serious except that it is not the DC resistance of the wire that matters, but the RF resistance, and this varies with frequency. The sensible recourse is therefore, once again, to minimise it; which means that tinned wire is forbidden, and the coil should be wound with enamelled copper or silver-plated wire. The wire diameter should also be reasonably large; except that this is in conflict with the need to keep the transformer dimensions small and avoid overlapping the turns. Some people like to stretch wire to remove the kinks before using it to wind coils. This practice should be avoided because it work-hardens the copper and increases its resistivity. Wire can be de-kinked without work-hardening by stroking it with a soft cloth or paper towel.

If the leakage reactance and winding resistance are kept small in relation to the secondary load resistance, then the secondary inductance, the self-capacitance, and the core losses, are effectively in parallel with the secondary load. This means that *most* of the transformer non-idealities can be lumped into a single impedance in parallel with the secondary winding, a simplification that permits straightforward and effective circuit optimisation. Note however, that the lumped component model breaks down completely at about twice the secondary network self-resonance frequency (the first *series* resonance of the parallel network). Self-resonance is associated with the length of the secondary winding wire; which means that, in order to achieve a wide bandwidth, the number of turns should be kept low. A compromise is necessary here; because the number of turns also dictates the secondary inductance and, as we shall see, insufficient inductance gives rise to a poor low-frequency response.

One component of the model that is not strictly amenable to the lumped-parameter approach is the 'self capacitance'. The self capacitance is largely a fiction, its purpose being to simulate a phase lag in the transformer output voltage that occurs at high frequencies. One cause of the HF phase error is that it takes a finite time for an electromagnetic wave to propagate along the winding wire, i.e., the coil is a transmission line. This transmission line is also dispersive, i.e., the propagation velocity varies with frequency but remains reasonably constant provided that the wire length is considerably less than one half-wavelength at the maximum frequency of operation. Hence there is an inherent inaccuracy in the standard model, but in general we will be able to cope with any noticeable discrepancy by considering the HF phase-shift as a separate phenomenon. One consequence, for example, is that the HF amplitude response of a transformer does not necessarily fall-off as rapidly as the 'self capacitance' would lead us to believe. This means that frequency-response calculations for phase-insensitive circuits such as RF ammeters and magnitude bridges can become more accurate when the self-capacitance is ignored [see Amplitude resp. of conventional and maxflat I transformers].

Note that the 'self-capacitance' is designated C_i'. The reason for the prime is that there will be some genuine parasitic capacitance across the transformer secondary network; due to the capacitance across the ends of the coil and between the lead wires; and due to the capacitance of the load resistor and any detector that might be connected. It also transpires that any parasitic effect that causes a phase shift in the transformer output will manifest itself as a contribution to the apparent self-capacitance (such effects can also cancel the actual self-capacitance and make the apparent self-capacitance negative). Hence the capacitance required for design purposes is:

$C_i = C_i'$ + true parallel capacitance + apparent capacitance not due to the transformer itself.

Finally, the core loss, being modelled as a resistance, appears to have no consequence except for a small reduction in our expectations for output voltage; but unfortunately, this loss also varies with frequency and so will affect the frequency response. It is therefore important to keep the core loss down, which implies that we should try to choose core materials of low initial permeability. We do not want the core permeability to be too low however, since this will make it difficult to obtain sufficient secondary inductance, and it will allow some of the magnetic field to escape from the core and give rise to leakage inductance. There is also, incidentally, a parallel resistive component due to dielectric loss (shown referred to the secondary); which is why it was stated earlier that plasticised PVC insulation should be avoided, and that the dielectric and coating of the primary coaxial-cable stub should be made from material that has a low Tanδ.

The author is aware of published articles on the subject of SWR bridge design that say that the current transformer secondary can be wound with hook-up wire (i.e., PVC insulated, tin-plated wire). Hopefully, from the foregoing discussion, it can be see that this is bad advice. PTFE-coated silver-plated wire can be used, but enamelled (soft) copper is better because the coating is thin, giving the greater conductor diameter for a given overall wire diameter.

6.2-3. Working models:

Many RF current transformer circuits in use and in the literature are based on the supposition that the simple ideal transformer model is adequate. This may be true if the transformer is to be used to make an RF ammeter to be operated over a limited frequency range, but it is a poor assumption if the transformer is to be used in a bridge intended to work over several octaves. If the transformer is suitably constructed however, it is possible to simplify the general model by assuming that all of the non-idealities can be represented as a network in parallel with the secondary winding. In that case, the working parameters are no longer strictly identifiable as secondary inductance, self-capacitance and loss resistance because they will have absorbed other non-idealities into their apparent values. The secondary inductance L_i, for example, may be adjusted to account for the fact that the permeability of the core material, and hence the A_L value, may vary slightly with frequency. The difference will not be great enough however, to warrant adding a prime or recurrent use of the word 'apparent' when discussing it.

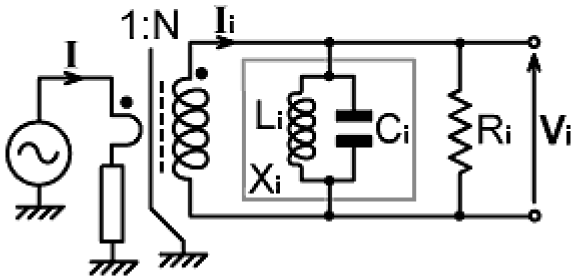
The point in representing all of the non-idealities as an impedance in parallel form is that the transformer input-output (transfer) relationship can be inverted. The impedance is then converted into an admittance that is readily separable into real and imaginary parts. The real part is the reciprocal mid-band transfer function, and the imaginary part contains all of the information relating to the frequency-response. This simplified quasi-empirical model is capable of describing the current transformer amplitude and phase performance to better than ±0.1% and ±0.1° over five or more octaves [A6.4].

The greatest non-ideality of the transformer is due to the finite secondary inductance. Hence this inductance can never be neglected in design calculations. Capacitance and resistance can however be neglected in certain circumstances. It was mentioned earlier that self-capacitance can often be ignored when designing circuits that discard phase information. It is also possible to devise passive compensation networks that cause a current transformer to behave as though its secondary reactance is purely inductive (to a good approximation) over a wide range of frequencies [A6.4]. We may also note, that if the auxiliary circuit is provided with some gain or sensitivity adjustment, there may be little point in including a resistive element to account for the fact that the transformer mid-band output voltage will be slightly lower than that of an ideal transformer. Hence, the complexity of the model can be adjusted according to circumstance.

6.2-4. Secondary reactance.

In practice, most of the non-ideality of a current transformer can be accounted for by presuming the existence of a reactance X_i connected in parallel with the secondary winding. This reactance is such that the phase of the output voltage leads the phase of the input current at low frequencies and lags at high frequencies, i.e., it is inductive at low frequencies and capacitive at high frequencies. Hence, in the lumped component model shown below, it is represented as an inductor in parallel with a capacitor. When designing wide-bandwidth measuring instruments, particularly bridges, it is necessary to quantify these additional components so that some kind of compensation can be applied.

It is easy to predict the low-frequency response because a good estimate for L_i can be had simply by measuring the inductance of the



secondary winding (L_{sec}). As mentioned previously, the effective value of L_i will be about 1% less than L_{sec}. L_i can also be estimated to within about ±20% by using the published A_L value for the magnetic core, i.e.,

$L_{sec} = A_L N^2$

The capacitance is a more problematic however. It is made up from several components: one being the stray capacitance due to the connecting leads and the capacitance of the secondary load resistor R_i; another being the semi-fictitious 'self-capacitance' that represents the propagation delay. It was mentioned before that there is a subtle difference between the phase lag due to a lumped capacitance and the phase lag due to a time delay, and the former will only serve as an approximation for the latter if the delay is small. To these basic contributors, we must also add the fact that mismatch of the primary transmission line, capacitive currents induced in the Faraday shield, and the inductance of the secondary load resistor, all make contributions to the apparent or 'effective' secondary capacitance. Hence it can be difficult to place a value on the effective capacitance, leading to extremely inconsistent results when traditional approaches to current-transformer bridge design are followed.

We will look into the subject of transformer phase response in considerable detail later, but before we can do that we must incorporate X_i into the mathematical model. The ampere-turns rule can still be applied, i.e. (using the definitions from the diagram above), $\mathbf{I}=N\mathbf{I}_i$ as before, but now $\mathbf{I}_i$ has become a fiction because X_i takes a bite out of it before it reaches the transformer terminals. We can still obtain an expression for the secondary voltage however, by presuming $\mathbf{I}_i$ to exist and by defining the secondary load as:

$\mathbf{Z}_i = R_i \,//\, \mathbf{j}X_i$

(where the symbol // should be read as: "in parallel with", and $\mathbf{a}//\mathbf{b}=\mathbf{ab}/(\mathbf{a}+\mathbf{b})$). Hence:

$\mathbf{V}_i = \mathbf{I}_i \mathbf{Z}_i$

and since $\mathbf{I}_i=\mathbf{I}/N$

$\mathbf{V}_i = \mathbf{I}\,\mathbf{Z}_i / N$	**4.1a**
or	
$\mathbf{V}_i = \mathbf{I}\,(R_i \,//\, \mathbf{j}X_i) / N$	**4.1b**
or	
$\mathbf{V}_i = \mathbf{I}\,(R_i \,//\, \mathbf{j}X_{Li} \,//\, \mathbf{j}X_{Ci}) / N$	**4.1c**

Note that when $X_{Li}+X_{Ci}=0$, then $X_{Li}//X_{Ci}\rightarrow\infty$., leaving only R_i as the secondary load. The secondary voltage only falls exactly into phase with the primary current at the secondary network 'pseudo-resonance' frequency, and there is a phase error everywhere else. This error can be kept small by making R_i small in relation to $|X_i|$ in the frequency range of interest, i.e., R_i is chosen to damp the pseudo-resonance to such an extent that a nearly-flat phase and frequency response is obtained. We cannot damp the resonance completely however, because that would require $R_i=0$ and hence no output, and so it is usual practice to adopt a low but finite value for R_i and deal with the residual errors in other ways.

6.2-5. Empirical efficiency factor.

Equations (**4.1a-c**) above generally give a good description of the overall shape of the transformer frequency-response curve, but are a little optimistic regarding the absolute output voltage obtained. The reason is that the transformer is not 100% efficient. In practice, the actual output voltage magnitude can be expected to be somewhere between 96 and 99% of that predicted by the 'ideal transformer with parallel reactance' model [see Current transformer efficiency factor].

The first question we should ask about this discrepancy is; "does it matter?", and the answer is generally "no". The point is that none of the transformer parameters can be predicted with absolute certainty because there are always engineering tolerances to consider. This means that it is always advisable to design circuits in such a way that they can be adjusted during calibration. If we design an RF ammeter to have (say) a ±20% adjustment in the meter series-resistor, or a bridge to have a ±20% adjustment in the ratio of the voltage-sampling network, then a model discrepancy of about 3% will go un-noticed and the instrument will still achieve its design performance targets. Hence there is usually no point in over-complicating the mathematics of the design process.

In the event that transformer efficiency is critical however, the shortfall can be accounted for by invoking an empirical coupling factor (k say) and using it to modify the predictions of equations (**4.1**). One way in which we might elect to use this approach is to rewrite equation (**4.1c**) as:

$\mathbf{V}_i = \mathbf{I}\, k\, (R_i \mathbin{//} \mathbf{j}X_{Li} \mathbin{//} \mathbf{j}X_{Ci}) / N$

where k has a value of about 0.97. This will do the trick, but it is unsatisfactory in two ways. Firstly, k is not directly identifiable as an electrical component, and this will be a nuisance in circuit simulations. Secondly, when k is multiplied into the impedance bracket, it will affect the resistance, the inductance and the capacitance. Although L_i and C_i are affected by the simplifications used to obtain a working model, they still remain closely associated with true physical parameters. They can also be measured in a manner that is unaffected by the choice of R_i (i.e, they have discrete identities). Hence it is more sensible to apply k in such a way that it only affects the apparent load resistance, i.e.:

$\mathbf{V}_i = \mathbf{I}\, (kR_i \mathbin{//} \mathbf{j}X_{Li} \mathbin{//} \mathbf{j}X_{Ci}) / N$	**5.1**

One consequence of allowing k to operate only on the load resistance is that it can be translated into a single electrical component. To do that, we note that the effective load resistance (R_{ik} say) is given by the expression:

$R_{ik} = k\, R_i$

and that we can achieve the same result by placing a resistance in parallel with R_i, i.e.:

$R_{ik} = R_i \mathbin{//} R_k$

Hence:

$k\, R_i = R_i \mathbin{//} R_k = R_i\, R_k / (R_i + R_k)$

i.e.:

$k = R_k / (R_i + R_k)$

which rearranges:

$k\, (R_i + R_k) = R_k$

$k\, R_i + k\, R_k = R_k$

$k\, R_i = R_k - k\, R_k = R_k\, (1 - k)$

$R_k = k\, R_i / (1 - k)$	**5.2**

Take, for example, a current transformer that has a secondary load resistance of 50Ω and a transfer efficiency of 97%. Then:

$R_k = 0.97\, R_i / 0.03 = 1617\Omega$

The output voltage of the transformer is, of course, given by:

$\mathbf{V}_i = \mathbf{I}\, (R_i \mathbin{//} R_k \mathbin{//} \mathbf{j}X_{Li} \mathbin{//} \mathbf{j}X_{Ci}) / N$	**5.3**

or, if there is an additional load, such as the input resistance of a detector:

$\mathbf{V}_i = \mathbf{I}\, (R_i \mathbin{//} R_k \mathbin{//} R_{Load} \mathbin{//} \mathbf{j}X_{Li} \mathbin{//} \mathbf{j}X_{Ci}) / N$	

The only thing that might appear to be unsatisfactory about the definition of R_k, as given in equation (**5.2**), is that the value of this lumped loss resistance seems to depend on R_i; and R_i is an external load resistance, not part of the transformer. It transpires however, that for transformers with low winding resistance, the efficiency k increases as the load resistance is reduced [see Current transformer efficiency factor]. This means that R_k is approximately constant, i.e., it is largely independent of R_i and therefore a true parameter of the transformer. This is reasonable of course, because R_k is mainly representative of the core loss and the dielectric loss referred to the secondary network. Incidentally, when a laboratory bridge is used to measure the secondary inductance of the transformer, the loss-balance adjustment expressed as a parallel resistance provides a first (but somewhat optimistic) estimate of R_k.

Now notice that a better figure for the transresistance (without the 'nominal') is given by:

$R_m = R_{ik} / N$

R_m (the mid-band transresistance) is typically about 2% smaller than $R_{m(nom)}$. Also, we can now define a complex input-output relationship, the transfer-impedance or ***transimpedance***:

$\mathbf{Z}_m = (R_{ik} // \mathbf{j}X_{Li} // \mathbf{j}X_{Ci}) / N$ [Volts per Amp]

By computing the transimpedance from fixed values of R_{ik}, L_i and C_i (the last of which can be positive or negative), and presuming that leakage inductance, winding resistance and propagation delay are sensibly minimised; it is possible to model the unloaded output voltage to an accuracy of somewhat better than 0.1%, over 4 or 5 octaves, using only four parameters.

6.2-6. Dimensionless transfer function.

Although the principal relationship is between the primary current and the secondary voltage, the output of current transformer is also related to the voltage across the load into which the primary current is being delivered. If the voltage across that load is **V**, and the impedance is **Z**, then its relationship to the primary current is given by Ohm's law:

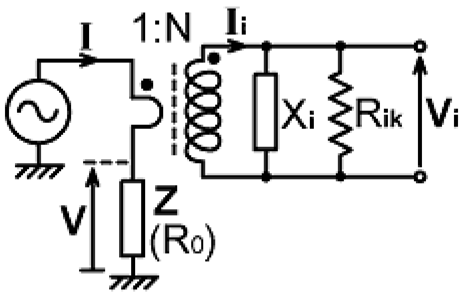
$\mathbf{I} = \mathbf{V} / \mathbf{Z}$

Substituting this into equation (**4.1a**) gives:

$\mathbf{V}_i = \mathbf{V}\,\mathbf{Z}_i / (N\,\mathbf{Z})$

(where $\mathbf{Z}_i = R_{ik} // \mathbf{j}X_i$). This allows us to define a dimensionless transfer function:

$\boldsymbol{\eta}_i = \mathbf{V}_i / \mathbf{V} = \mathbf{Z}_i / (N\,\mathbf{Z}) = \mathbf{Z}_m / \mathbf{Z}$	**6.1**



(where $\boldsymbol{\eta}$ is Greek lower case"eta", commonly used to represent ratios, in this case **bold** because it is complex).

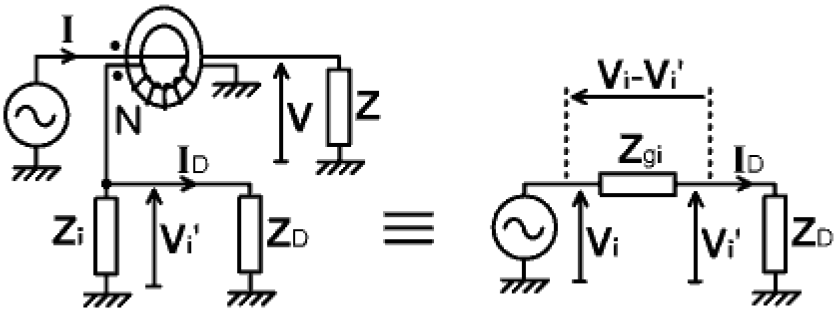
The balance point for a bridge is independent of the voltage or current associated with the primary load. Hence, when designing bridges, the use of dimensionless transfer functions preserves this independence. The design process then reduces to the matter of defining transfer functions for the current and voltage sampling networks and setting them to be equal when $\mathbf{Z}=R_0$. In that case, the transfer function for the current sampling network becomes:

$\boldsymbol{\eta}_0 = \mathbf{V}_i / \mathbf{V} = \mathbf{Z}_i / (N\,R_0)$

6.2-7. Output impedance.

One of the main applications of the current transformer is that of driving a detector. Thus there are many situations in which the input impedance of a detector forms part of the secondary load. Note that this is not the case when the transformer is incorporated into an impedance bridge and the bridge is at balance; because the outputs of a current sampling network and a voltage sampling network are placed in series opposition and both networks are unloaded when the balance point is reached (no detector current is drawn because the two voltages cancel). It is an issue in SWR bridges however, because SWR measurement involves taking detector readings from bridges that are out of balance, and it is an issue in magnitude and other scalar-addition bridges, because the sampling network outputs are rectified before summation. The easiest way to take care of this matter is to be aware of the output impedances of all of the sampling networks used, and so a derivation for the output impedance of a current transformer will be given here.

The output impedance of a network is defined as the impedance that, when placed in series with a hypothetical perfect generator, accounts for the drop in output voltage that occurs when the network is loaded. Shown below is a representation of a current transformer network feeding a detector that has an input impedance $\mathbf{Z}_D$. If the load $\mathbf{Z}_D$ is removed, the output of the network is $\mathbf{V}_i$, but when the load is connected the output sags slightly (and may be shifted in phase) to a new voltage $\mathbf{V}_i'$. This situation is modelled on the right as a perfect generator with an output $\mathbf{V}_i$ in series with an impedance $\mathbf{Z}_{gi}$, the latter being the output impedance we wish to determine.



Using the definitions given in the diagram:

$\mathbf{Z}_{gi} = (\mathbf{V}_i - \mathbf{V}_i') / \mathbf{I}_D$

where:

$\mathbf{I}_D = \mathbf{V}_i' / \mathbf{Z}_D$

Hence, combining these two equations:

$\mathbf{Z}_{gi} = \mathbf{Z}_D (\mathbf{V}_i - \mathbf{V}_i') / \mathbf{V}_i'$

$\quad = \mathbf{Z}_D [(\mathbf{V}_i / \mathbf{V}_i') - 1]$ . . . . . . (**7.1**)

From equation (**4.1a**) given earlier:

$\mathbf{V}_i = \mathbf{I}\, \mathbf{Z}_i / N$

and by considering $\mathbf{Z}_D$ as part of the secondary load:

$\mathbf{V}_i' = \mathbf{I} (\mathbf{Z}_i // \mathbf{Z}_D) / N$

Hence

$\mathbf{V}_i / \mathbf{V}_i' = [\mathbf{I}\, \mathbf{Z}_i / N] / [\mathbf{I} (\mathbf{Z}_i // \mathbf{Z}_D) / N]$

$\quad = [(1/\mathbf{Z}_i) + (1/\mathbf{Z}_D)] / (1/\mathbf{Z}_i)$

$\mathbf{V}_i / \mathbf{V}_i' = 1 + (\mathbf{Z}_i / \mathbf{Z}_D)$

Substituting this into (**7.1**) gives:

$\mathbf{Z}_{gi} = \mathbf{Z}_D [1 + (\mathbf{Z}_i / \mathbf{Z}_D) - 1]$

i.e.:

$\boxed{\mathbf{Z}_{gi} = \mathbf{Z}_i}$

The output impedance of a current transformer is the same as the impedance shunting the secondary winding. Observe also, that if the transformer is operating in the middle of its passband and the phase error is no more that a few degrees ($|X_i| >> R_i$), the output impedance as a measure of detector driving capability is essentially the same as the load resistance R_i (neglecting losses).

The input resistance of a simple diode half-wave detector was discussed in section **6-9** and shown to be approximately half the detector load resistance. Thus, for example, a detector connected to a meter of 10KΩ resistance will have an input resistance of about 5KΩ. If the detector is connected to a current transformer having a secondary load of (say) 50Ω, then the effective load becomes 50//5000 = 49.505Ω. Hence, if the detector input resistance is 100 times the secondary load resistance, the detector will cause the transformer output to drop by no more than 1%.

The derivation given above incidentally, contains a hidden approximation. This is due to the assumption that the primary current **I** will not change when the secondary load is changed. In fact, since reducing the secondary load impedance will cause the input impedance of the transformer to go down, the current **I** will increase very slightly when the secondary is loaded. Hence the output impedance of the network is slightly less than indicated by the derivation. If however, we stick to the practice of making the transformer input impedance less than 1% of the primary load, and make the detector input resistance at least 10 and preferably 100 times greater than the secondary load resistance, then the deviation will be at the 0.1 to 0.01% level and can safely be neglected.

There is also, of course, an approximation inherent in neglecting the transformer losses. This can be overcome by replacing R_i with $R_{ik}=R_i//R_k$ (losses reduce the output impedance slightly) but it is rarely necessary to consider such subtleties when determining load driving capability.

6.2-8. Choosing the turns ratio:

The principal input-output relationship for a current transformer was given earlier as:

$\mathbf{V}_i = \mathbf{I}\ \mathbf{Z}_i / N$

If we just want to know the sensitivity of the device, we can ignore the phase information in this expression by taking the magnitudes of both sides:

$|\mathbf{V}_i| = |\mathbf{I}|\ |\mathbf{Z}_i| / N$

We can also argue that, if the secondary load resistance R_i is low enough to swamp the transformer reactance, then phase errors will be small and $|\mathbf{Z}_i|$ will be approximately equal to R_i. Hence the mid-band sensitivity of a current transformer (neglecting losses) is given by the expression:

$\lvert\mathbf{V}_i\rvert = \lvert\mathbf{I}\rvert\ R_i / N$

or, using scalar notation:

$V_i = I\ R_i / N$

This tells us that the turns ratio N is dictated by the required output voltage, the secondary load resistance, and the current to be measured.

If a sampling network is to drive a simple passive indicating device such as a half-wave rectifier; good detector linearity can be obtained with an output voltage of about 7V RMS, which gives 10V after rectification. Output voltages greater than that give improved linearity of course, but when using signal diodes, it is advisable to allow a large safety margin with regard to reverse breakdown. Lower voltages are acceptable when the actual meter reading is unimportant, i.e., for null indicators; but if the maximum voltage after rectification is much below 1V, the linearity will be poor and, due to the diode forward threshold, nulls will become broad and difficult to locate. Hence, for our purposes, a reasonable design window for current transformer output is 1 to 10V after rectification, i.e., $|\mathbf{V}_i|$ between about 0.7 and 7V RMS. If the indicating device is a (readily available) 100µA meter, then padding its resistance to 100KΩ gives a sensitivity of 10V DC for full-scale deflection (FSD) with a detector input resistance of 50KΩ; and padding it to10KΩ gives 1V for FSD and an input resistance of 5KΩ.

It is common practice to load the secondary of a current transformer with a resistance of about 50Ω. This choice usually permits a transformer bandwidth of 5 octaves or more, gives sensible turns ratios for useful output voltages, and enables detectors of 5KΩ input resistance to be connected with only about 1% drop in output. There is also the advantage that a 50Ω load resistor can be replaced by an arbitrary length of coaxial cable with a terminating resistor at the far end, a possibility that facilitates testing and allows the detector to be placed some distance from the transformer.

The table below shows the number of turns required in order to obtain a given output voltage for a given primary current when the transformer secondary is loaded with 50Ω. The left-most column shows the generator output power required to produce the current when the generator is working into a 50Ω load. The candidate output voltages correspond to 10V, 3.16V (i.e., √10V), and 1V after rectification (neglecting diode forward voltage drop). The actual number of turns used must, of course, be an integer (i.e., a whole number), and so for a practical transformer the number must be rounded up or down.

Table 6-16#1. Relationship between current transformer output voltage and no. of turns.

Generator output power (=P)	Primary current $\lvert\mathbf{I}\rvert=\sqrt{(P/50)}$	Turns for 7.07V $N=50\lvert\mathbf{I}\rvert/7.07$	Turns for 2.24V $N=50\lvert\mathbf{I}\rvert/2.24$	Turns for 0.71V $N=50\lvert\mathbf{I}\rvert/0.71$
1500 W	5.48 A	38.7	122.5	387.3
1200 W	4.90 A	34.6	109.5	346.4
1000 W	4.47 A	31.6	100.0	316.2
800 W	4.00 A	28.3	89.4	282.8
500 W	3.16 A	22.4	70.7	223.6

400 W	2.83 A	20.0	63.2	200.0
200 W	2.00 A	14.1	44.7	141.4
100 W	1.41 A	10.0	31.6	100.0
50 W	1.00 A	7.1	22.4	70.7
10 W	0.45 A	3.2	10.0	31.6

From the table it would appear that the required turns ratio varies between about 3 and 400 for a typical range of sensitivity requirements and a 50Ω secondary load, the large numbers being associated with instruments of low sensitivity and vice versa. In practice however, a large number of turns implies an excessive length of winding wire and a small wire diameter. The result will be a large phase-lag at high frequencies and high RF resistance, the latter causing deviation from the 'ideal transformer with parallel impedance' model. Hence it is better to obtain low sensitivity (i.e., low transresistance) by using a low number of turns and a value of secondary load resistance somewhat less than 50Ω.

A commonly accepted practice in determining the maximum number if turns is to limit the wire length to less than 0.1λ at the highest frequency of operation (λ being the electrical wavelength). This rule of thumb is somewhat arbitrary in relation to the physics of what is going on inside the transformer, and windings approaching that length will certainly not give good phase performance from the point of view of the bridge designer; but it perhaps represents a limit of awfulness beyond which compensation ceases to be feasible and so may inform our initial considerations.

The wire-length for a given number of turns can be computed from core dimensions, and (assuming a velocity factor v/c of about 0.83) the 0.1λ rule suggests that it should be no longer than about 1.2m for a transformer designed for 30MHz (about 66 turns on a ½" core assuming a length per turn of about 18mm), and 67cm for 54MHz (about 37 turns on a ½" core).

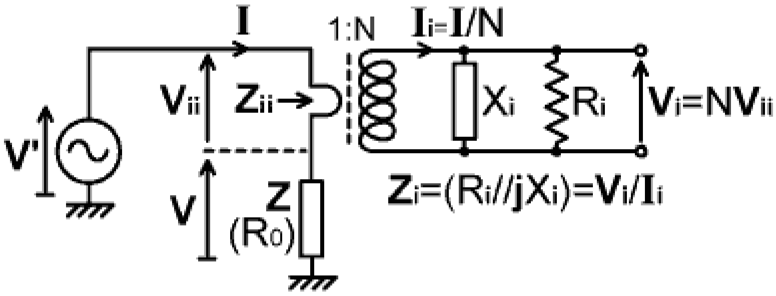
Although putting too many turns on the transformer secondary is a bad idea, there is also a limit to the *minimum* number of turns that can be used because insufficient secondary inductance will lead to a poor low-frequency response. If the required sensitivity cannot be reconciled with the need to obtain a reasonable amount of inductance, increasing the secondary load resistance will not help because it will narrow the transformer bandwidth. It is however possible to increase the number of turns in the primary by using thin coaxial cable and passing it two or more times through the core (see **Current transformer efficiency factor**, fig. 6).

We will look into the secondary inductance and winding-length issues in detail shortly, but for now it will be useful to remember that small current transformers intended for HF radio applications (up to 30MHz) tend to have secondary turns numbers in the 10 to 40 range.

6.2-9. Insertion impedance and insertion loss:

There is a little more to the design process that just considering the output voltage and frequency response because the transformer has an input impedance and therefore causes a voltage drop in the transmission-line into which it is inserted. The transformer also abstracts a certain amount of energy, and we need to consider what happens to the resulting heat.

In order to calculate the insertion loss due to a current transformer, and hence the power rating of the secondary load, consider the circuit in the manner shown below:



Using the definitions in the diagram, the input impedance or 'insertion impedance' of the primary is $\mathbf{Z}_{ii}=\mathbf{V}_{ii}/\mathbf{I}$. Now note that $\mathbf{V}_{ii}=\mathbf{V}_i/N$ and $\mathbf{I}=N\mathbf{I}_i$. Hence:

$\mathbf{Z}_{ii} = (\mathbf{V}_i / N) / (N \mathbf{I}_i)$

i.e.,

$\mathbf{Z}_{ii} = \mathbf{Z}_i / N^2$

As is the case for an ideal transformer having a closed magnetic circuit, and as is true to a good approximation for practical transformers; the input impedance is the secondary load impedance divided by the square of the turns ratio.

If the current transformer is used as part of a monitoring device in a power transmission system, then we can work out the operating insertion loss on the basis that the primary load impedance will be adjusted to be a pure resistance R_0 (i.e., the transmitter cannot be turned-up to full power until this adjustment has been carried out). We may also observe that maximum energy will be absorbed when the reactance X_i is large in relation to the secondary load resistance R_i (i.e., in the middle of the passband); in which case the insertion impedance of the transformer will be predominantly resistive. Hence we can deduce the worst-case insertion loss on the basis of a nominal mid-band 'insertion resistance' R_{ii}, where:

$R_{ii} = R_i / N^2$

The ratio of primary load voltage to generator voltage for the system depicted above is $\mathbf{V}/\mathbf{V}'$, this being dictated by the potential divider network formed by R_{ii} in series with R_0, i.e.,

$\mathbf{V} / \mathbf{V}' = \mathbf{V} / (\mathbf{V}_{ii} + \mathbf{V}) = \mathbf{I} R_{ii} / \mathbf{I} (R_{ii} + R_0)$

The primary current is eliminated giving:

$\mathbf{V} / \mathbf{V}' = R_{ii} / (R_{ii} + R_0)$

Hence the mid-band insertion loss in dB is:

$\text{Insertion loss / dB} = -20 \text{ Log}[R_{ii} / (R_{ii} + R_0)]$

The worst-case power dissipated in the secondary load resistance (i.e., assuming no losses in the transformer) is equal to $|\mathbf{I}|^2 R_{ii}$, i.e.,

$P_i = |\mathbf{I}|^2 R_i / N^2$

Note that if the secondary winding is split or tapped and the two sections are terminated individually, the power in the total secondary load resistance is divided between the two load resistors.

Example 1: For a current transformer with 40 turns on the secondary, and a 50Ω secondary load, the nominal mid-band insertion resistance, (R_i/N^2), is 0.03125Ω. This corresponds to an insertion loss of 0.005dB, i.e., -20Log[50/(50.03125)], when the generator (transmitter) is working into a 50Ω load. If the generator is delivering 1.5KW, the primary current, √(P/R), is 5.48A. The secondary current is 5.48/40=137mA, and the secondary voltage is 0.137×50=6.85V RMS. The power dissipated in the secondary load resistor is 0.137×6.85=0.94W. The secondary load can therefore be constructed by placing two non-inductive 100Ω resistors of at least 0.5W power rating in parallel.

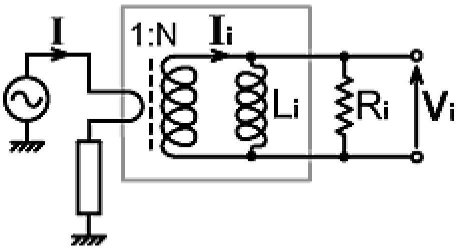
Example 2: For a current transformer with 10 turns on the secondary and a 50Ω secondary load, the nominal insertion resistance is $50/10^2$=0.5Ω. This corresponds to a maximum (mid-band) insertion loss of -20Log(50/50.5)=0.09dB when the generator is working into a 50Ω load. If the generator is delivering 100W, the primary current is 1.414A, and the secondary current is 141.4mA. The secondary voltage is

0.1414×50=7.07V RMS, and the power dissipated in the secondary load resistor is 1W.

1950s vintage RF ammeter: Used to measure the antenna current of an HF maritime broadcast transmitter. The transformer primary is a silver-plated brass rod with a silver-plated Faraday shield. The magnetic core is an un-coated iron-powder toroid. The secondary load resistor is connected directly across the coil terminals. A Germanium detector diode and a 1nF smoothing capacitor are included in the box.

6.2 Current Transformers: Part 2.

10. Low-frequency performance:
The effective secondary parallel reactance of a current transformer in the low-frequency region of its passband is dominated by the inductance of the secondary winding. Hence, for the purpose of calculating the low-frequency response, the model can be simplified as shown on the right.

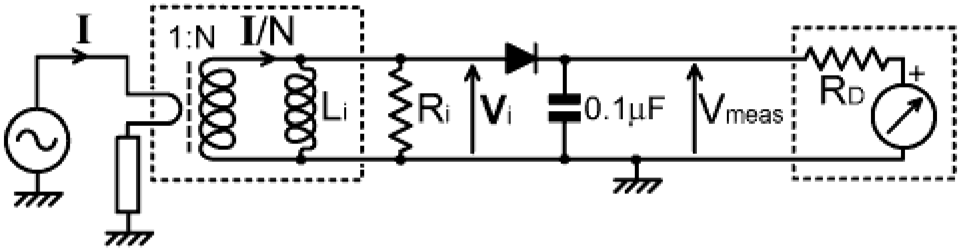


Here the capacitance has been ignored, and the inductance L_i should be taken to be about 1 or 2% less than the measured inductance of the secondary coil. The relationship between primary current and secondary voltage (neglecting losses) given earlier as equation (**4.1**) is now reduced to:

$$\mathbf{V}_i = \mathbf{I}\ (R_i\ //\ \mathbf{j}X_{Li})\ /\ N$$

The obvious implication is that there will be significant phase and amplitude errors in the output unless $X_{Li} >> R_i$. In fact, the lower -3dB bandwidth limit occurs when $X_{Li} = R_i$, with an attendant phase error of +45°.

Phase and magnitude errors in the current sampling network of a bridge will compromise the accuracy of the balance condition. We will see in later sections however, that compensation can be achieved by modifying the voltage sampling network so that its output has the same frequency response as that of the current sampling network. Such compensation however, does not correct for the low-frequency reduction in output voltage, and this has implications with regard to the sensitivity of bridges, and the accuracy of current-measuring instruments. It is therefore principally the desire for a flat amplitude response that dictates the minimum value of inductance that can be tolerated, and we can understand this issue by considering the simple RF ammeter circuit shown below:



Here the detector is sensitive only to the magnitude of the output voltage, and the reading on the meter (assuming a scale calibrated to allow for diode forward voltage drop) is given by:

$V_{meas} = (\sqrt{2})\ |\mathbf{V}_i|$

where:

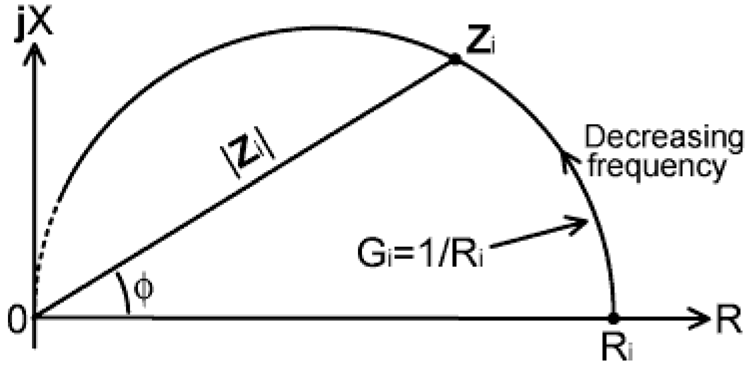
$|\mathbf{V}_i| = |\mathbf{I}|\ |(\mathbf{j}X_{Li}\ //\ R_i)|\ /\ N$

i.e.,

$$V_{meas} = (\sqrt{2})\ |\mathbf{I}|\ |(\mathbf{j}X_{Li}\ //\ R_i)|\ /\ N$$

The measured output (i.e., the sensitivity) is therefore always proportional to the magnitude of the secondary load impedance, which (neglecting losses and the detector input resistance) consists of the secondary winding reactance in parallel with the load resistance.

The effect of placing an inductance in parallel with an impedance [discussed in Impedance Matching, section **5-7**] is to move the resultant impedance anti-clockwise around a circle of constant conductance as the inductive reactance decreases. The reactance $X_{Li}=2\pi fL_i$, of course, decreases as the frequency decreases, and the constant conductance G_i is equal to $1/R_i$. We can therefore visualise the process using the Z-plane diagram below:



Where $\mathbf{Z}_i = (\mathbf{j}X_{Li} \,//\, R_i)$

An expression for the magnitude of an impedance in its parallel form was given in [**AC Theory, Section 18, eqation 18.2**]. Using the present notation it becomes:

$|\mathbf{Z}_i| = | R_i X_{Li} / \sqrt{(R_i^2 + X_{Li}^2)} |$

At high frequencies, X_{Li} becomes very large and its contribution to $\mathbf{Z}_i$ becomes correspondingly small. In this capacitance-free model therefore, the magnitude of $\mathbf{Z}_i$ can be considered to be equal to R_i at high frequencies, i.e., as $X_{Li}\to\infty$, $|\mathbf{Z}_i|\to R_i$. Consequently, since the measured voltage for a given input current is proportional to $|\mathbf{Z}_i|$, we can define a dimensionless amplitude-response function for the system as:

$\eta_{LF} = |\mathbf{Z}_i| / R_i = |X_{Li}| / \sqrt{(R_i^2 + X_{Li}^2)}$

(η is "eta", this time un-bold because the quantity it represents is scalar). The reason for deriving this expression, is that we can use it to obtain the minimum amount of inductance required in order to keep the drop in meter sensitivity within acceptable limits at the lowest frequency of operation. All we need to do is rearrange it until we have X_{Li} expressed in terms of R_i and η_{LF}, as follows:

$\eta_{LF}^2 = X_{Li}^2 / (R_i^2 + X_{Li}^2)$

$\eta_{LF}^2 (R_i^2 + X_{Li}^2) = X_{Li}^2$

$X_{Li}^2 - \eta_{LF}^2 X_{Li}^2 = \eta_{LF}^2 R_i^2$

$X_{Li}^2 (1 - \eta_{LF}^2) = \eta_{LF}^2 R_i^2$

$X_{Li}^2 = R_i^2 \eta_{LF}^2 / (1 - \eta_{LF}^2)$

Now taking the square root to obtain X_{Li}, we note that we only want the positive result (i.e., the magnitude of X_{Li}), and so:

$|X_{Li}| = R_i \eta_{LF} / \sqrt{(1 - \eta_{LF}^2)}$

but since X_{Li} is an inductive reactance, we know it will be positive and so we can dispense with the magnitude symbol in this instance (but not in general) to obtain:

$X_{Li} = R_i \eta_{LF} / \sqrt{(1 - \eta_{LF}^2)}$

The results for some possible design values for η_{LF} are tabulated below:

Table 10.1. Current transformer, low-frequency desensitisation and phase error.

Loss in % $=100(1-\eta_{LF})$	Loss in dB $=-20\mathrm{Log}(\eta_{fLF})$	η_{LF} $=\lvert\mathbf{Z}_i\rvert/R_i$	X_{Li} $=R_i\eta_{LF}/\sqrt{(1-\eta_{LF}^2)}$	φ =Arcan(R_i/X_i)	Required L_i for f_{min}=1.6MHz, R_i=50Ω
1 %	0.09 dB	0.99	7.02 R_i	8.1°	34.9μH
5 %	0.45 dB	0.95	3.04 R_i	18.2°	15.1μH
10 %	0.92 dB	0.90	2.06 R_i	25.8°	10.2μH
11%	1 dB	0.891	1.97 R_i	27°	9.8μH
29%	3dB	0.707	R_i	45°	5.0μH
50%	6 dB	0.5	0.58 R_i	60°	2.9μH

The figure $X_{Li}\geq 7R_i$ for 1% max. error is corroborated by ref [**The ARRL Antenna Book**, 19th edition, ARRL publ, 2000.
ISBN: 0-87259-804-7. Bridge types p27.4].

The data in the table tell us that if we want a current reading at the lowest frequency of operation (f_{min}) to be within 1% of a high-end reading of the same current, then X_{Li} must be at least 7 times larger than R_i at the lowest frequency. If we want agreement within 5%, then X_{Li} must be at least 3 times larger than R_i, and so on. The more stringent design requirements at the top of the table are appropriate for direct-reading RF ammeters, where scale accuracy is important; but for the design of RF bridges, where we are often in a position to turn up the generator level if the bridge sensitivity starts to fall, a reduction in sensitivity of 3dB, or even 6dB, may be perfectly acceptable.

Also shown in the table are the phase errors associated with the various design criteria. Notice that even when the amplitude is controlled to within 1%, the phase error at the minimum frequency is 8°. Underhill & Lewis [**43**] recommend that the maximum acceptable error for an impedance matching system should be ±7° (1.2:1 SWR), and so a large secondary inductance, notwithstanding the propagation-delay issue, is not a way of avoiding the need for phase compensation when designing bridges. We might, of course, decide to use a low value of load resistance *and* a large inductance; but that implies a low output voltage (i.e., insensitivity), which is fine when monitoring a transmitter producing kilowatts, but not so good when trying to tune an antenna using a few watts.

For those interested in designing accurate RF ammeters, note that using a large secondary inductance is neither the only, nor necessarily the best, way of obtaining a flat frequency response. By the inclusion of a capacitor, the secondary loading network can be modified to produce an output that is flat within 1% over the 1.6 to 30 MHz range (or greater) using an inductance of around 10 µH. This configuration, which does not appear to have been reported elsewhere, is referred to in these documents as the Maximally-Flat Current Transformer [see article of that name].

Ref:
[43] **"Automatic Tuning of Antennae"**. M J **Underhill** [G3LHZ] and P A **Lewis**.
SERT Journal, Vol 8, Sept 1974, p183-184.
Gives criteria for achieving 1.2:1 SWR, i.e., $45 \le R \le 56\Omega$, $17.5 \le G \le 22.5$ mS, $-7° \le \varphi \le +7°$.

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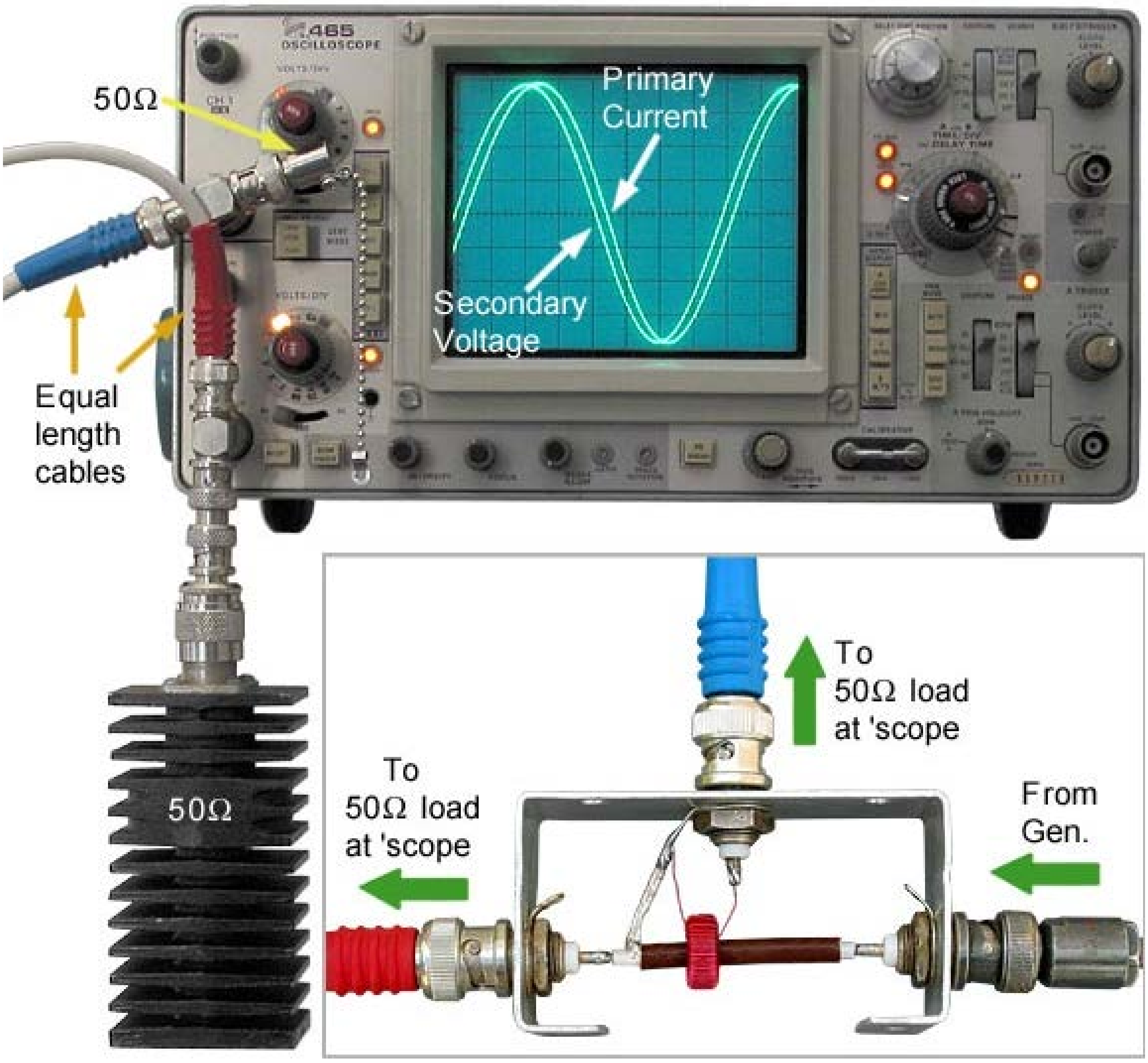
11. LF phase-error demonstration.

A simple technique for demonstrating current-transformer low-frequency phase-error is illustrated below. It uses a method that works well at low frequencies (1.6 - 3 MHz) but can give misleading results at higher frequencies unless it can be shown that the oscilloscope Y-amplifiers have identical propagation delay and that the delay does not vary with the settings of the gain controls. In this case a waveform with the same phase as the primary current is obtained by measuring the voltage across a 50 Ω load resistor that terminates the generator. When this is compared with the voltage appearing across a 50 Ω resistor terminating the secondary winding, a phase difference is seen on the oscilloscope screen. By adjusting the Y-amplifier gain and shift controls, both waveforms can be made to have exactly the same height and be equally displaced about the central horizontal scale on the graticule. The phase difference between the two waves is then:

$$\Delta\varphi = \frac{\text{Horizontal separation between waveforms}}{\text{Horizontal length of one cycle}} \times 360°$$



In the example above, the current transformer primary is a stub of URM108 (PTFE-Silver, 50Ω) cable, and the secondary is 61 turns of 36swg wire (0.225mm diameter including insulation) on an Amidon T50-2 (red) core. The published A_L (inductance factor) for this core is 4.9nH/turns², giving a nominal secondary inductance ($A_L N^2$) of 18.2μH (± about 20%), but the measured inductance at 1.5915MHz (10^7 radians/sec) was 19.9±0.5μH. The phase-difference measurement was made at 1.6MHz, at which frequency the reactance of the coil (2πfL) is +200±5Ω. The situation shown on the oscilloscope screen is therefore $X_{Li}=4R_i$.

The expected phase angle is:

Arctan(50/200±5) = 14.04±0.34°

Using the horizontal shift, timebase, and trigger level controls, the oscilloscope display was manipulated so that one cycle of the waveform was 10cm long. The distance between the zero-crossings of the two waves was then found to be 0.4 ±0.05cm. The measured phase difference is therefore:

360 × 0.4 / 10 = 14.40 ±1.8°

Which agrees with the calculated value.

On a practical point, notice that oscilloscope probes were not used, and that the signals were taken along 50Ω cables of equal-length and identical dielectric (for equal time-delay) and terminated at BNC T-pieces on the oscilloscope front panel. The problem with probes is that there will be capacitive coupling between them, and this gives rise to phase errors. The large transmitter terminator is shown hanging down from the front-panel; but a length of cable *after* the T-piece, so that the load can be placed more conveniently, is of no electrical consequence. The generator was a Kenwood TS430S radio transceiver with plug 10 removed from the RF module to give 1.6 to 30MHz transmitter coverage. The output level during the measurement was about 16Vp-p (5.7V RMS, 0.64W in 50Ω). No attenuator was used between the main transmitter line and the oscilloscope because it is important that both sampling points have the same input impedance (so that both cables are equally mismatched). For the instrument shown, both Y-amplifier inputs are 1MΩ // 20pF, which is typical.

12. Transformer core selection.

Having examined the basic design considerations for current transformers, we now turn our attention to the problem of choosing transformer cores from manufacturer's data. Here it will be assumed that we are primarily interested in designing current transformers for bridges, and a maximum low-frequency desensitisation of 3dB is acceptable; in which case the minimum secondary reactance must be no less than the load resistance. If the minimum frequency of operation is (say) 1.8MHz, and we intend to follow the normal practice of terminating the transformer with a resistor of about 50Ω, then the minimum secondary inductance $L_i = X_{Li}/2\pi f = 4.42\mu H$. With this figure in mind, we may trawl the catalogues looking for cores that will fit reasonably tightly over a stub of coaxial cable, and which have A_L (inductance / turns²) values that permit us to obtain a transformer ratio appropriate for our purpose.

Amidon supplies Micrometals and Fair-rite cores in small quantities (see also: links page for other sources). Hence, although suitable cores are available from various manufacturers, the use of Amidon products is convenient for private experimenters. Using the Amidon catalogue [**38**], and supplementary data from the Micrometals and Fair-rite websites, it transpires that the choice of core is remarkably limited once the frequency range and the hole diameter have been specified.

Ref: [38] **Amidon Associates Inc.** (Technical data book) Jan 2000.
Technical data for iron powder and ferrite cores, including A_L values, frequency ranges, wire packing tables, Q curves, etc. Maximum flux density calculations and recommendations: p1-35. Information is also available online from: **www.amidoncorp.com** .

The point is that we need the core to be a tight fit on the cable in order to minimise leakage inductance and magnetic path-length; and so once a cable diameter has been chosen, the core hole diameter is the next size up that allows room for the secondary winding. The choice of core material is then that which gives sufficient secondary inductance with the required number of turns. The relevant information is summarised below:

Table 16.3. 50Ω PTFE Coaxial Cable data.

Type	Overall dia. / mm	Jacket *	Dielectric	Max V_{RMS} / KV	Capacitance/m C_0 / pF/m**	Velocity factor
UR M72	4.5	FEP	PTFE	1.4	94	0.72
UR M102	9.7	FEP	PTFE	3.5	96	0.70
UR M107	9.0	FEP	PTFE	3.5	96	0.70
UR M108	4.5	FEP	PTFE	1.4	94	0.72
UR M109	2.45	FEP	PTFE	0.7	93	0.72
UR M110	1.8	FEP	PTFE	0.35	92	0.72
RG-142	4.95	PTFE / FEP	PTFE	1.4	95.8	0.695
RG-303	4.32	PTFE	PTFE	1.4	95.8	0.695
RG-316	2.49	PTFE / FEP	PTFE	0.9	95.1	0.695
RG-393	9.91	PTFE	PTFE	5.0	96.5	0.695
RG-400	4.95	PTFE	PTFE	1.9	96.5	0.695

Sources:
Uniradio Metric series data from: **BICC Cableselector E15**. PTFE Coaxial Cables. July 1979.
American Radio Guide series data taken from: **The ARRL Antenna Book**, 19th edition, ARRL publ, 2000. ISBN: 0-87259-804-7. Coaxial cable data p24-19.
* Coating materials vary / options exist - check manufacturer's data. NEVER use PVC in RF applications or where high temperatures may occur.
FEP = Fluorinated Ethylene Polypropylene. $\varepsilon_r' = 2.1$ (non-polar).
** Capacitance per unit length may vary depending on manufacturer. Inductance per unit length: $L_0 = 50^2 C_0$.

Table 16.4. Toroidal Core Dimensions:

Core	Outside dia. D /mm	Hole dia. d /mm	Thickness h /mm	Mean path l_e / cm	Core area A_e / mm^2	Turn length* / mm
T-25	6.35	3.05	2.44	1.5	4.2	
T-30	7.80	3.83	3.25	1.83	6.5	
T-37	9.53	5.21	3.25	2.32	7.0	
T-44	11.18	5.82	4.04	2.67	10.7	
T-50	12.70	7.62	4.83	3.20	12.1	≈18.7
T-68	17.53	9.40	4.83	4.24	19.6	≈21.8
T-80	20.19	12.57	6.35	5.15	24.2	
FT-23	5.84	3.05	1.52	1.34	2.1	
FT-37	9.53	4.75	3.18	2.15	7.6	
FT-50	12.70	7.14	4.78	3.02	13.3	≈19.1
FT-50A	12.70	7.92	6.35	3.68	15.2	≈21.5
FT-50B	12.70	7.92	12.7	3.18	30.3	≈34.2
FT-82	20.96	13.11	6.35	5.26	24.6	

*Wire length required for 1 turn estimated as ≈ D - d + 2h + 4mm

Table 16.5. Maximum number of turns in single-layer winding:

AWG	Wire dia / mm	T-25 (3.05 ID)	T-30 (3.83 ID)	T-37 (5.21ID)	T-44 (5.82 ID)	T-50 (7.62 ID)	T-68 (9.40 ID)	T-80 (12.6 ID)
18	1.024	4	5	9	10	16	21	30
20	0.810	5	7	12	15	21	28	39
22	0.643	7	11	17	20	28	36	51
24	0.511	11	15	23	27	37	47	66
26	0.404	15	21	31	35	49	61	84
28	0.320	21	28	41	46	63	79	108
30	0.254	28	37	53	60	81	101	137
32	0.201	37	48	67	76	103	127	172
34	0.160	48	62	87	97	131	162	219
36	0.127	62	78	110	124	166	205	276
38	0.099	79	101	140	157	210	257	347
40	0.079	101	129	177	199	265	325	438

Table 16.6. Core Materials:

Core material	Type	Initial permeability. μ_i	TC* 20-70°C / ppm/°C	Tuned circuit freq. range / MHz	Broadband xformer freq. range / MHz	Available sizes.
Carbonyl Iron	2 (red)	10	95	2 - 30	No data**	T-25, 30, 37, 44, 50, 68, 80
Carbonyl Iron	1 (blue)	20	280	0.5 - 5	No data	T-25, 30, 37, 44, 50, 68, 80
Carbonyl Iron	15 (red/wh)	25	190	0.1 - 2	No data	T-25, 30, 37, 44, 50, 68, 80

	3 (grey)	35	370	0.05 - 0.5	No data**	
Ni-Zn Ferrite	67	40	1300	10 - 180	200 - 1000	FT-23, 37, 50, 50A, 50B, 82
	61	125	1500	0.2 - 10	10-200	
	43	850	12500	0.01 - 1	1 - 50	

* Temperature coefficient of initial permeability. These figures are approximate (see manufacturer's graphs for more accurate figures at the temperature of interest).
** No data in catalogue, but known to be useful at least over the 1.8 - 30MHz range (Broadband transformer requirements are less stringent than for high-Q inductors).

Table 16.7. Iron powder cores, A_L Values / nH/turns², ±20%

Type	T-25	T-30	T-37	T-44	T-50	T-68	T-80
1 (μ_i=20)	7.0	8.5	8.0	10.5	10.0	11.5	11.5
2 (μ_i=10)	3.4	4.3	4.0	5.2	4.9	5.7	5.5
3 (μ_i=35)	10.0	14.0	12.0	18.0	17.5	19.5	18.0
15 (μ_i=25)	8.5	9.3	9.0	16.0	13.5	18.0	17.0

Table 16.8. Ferrite cores, A_L Values / nH/turns², ±25%

Type	FT-23	FT-37	FT-50	FT-50A	FT-50B	FT-82
43 (μ_i=850)	188	420	523	570	1140	557
61 (μ_i=125)	24.8	55.3	69	75	150	73.3
67 (μ_i=40)	7.8	17.7	22	24	48	22.4

Let us propose, at this point, that it has been decided that the current transformer will be fitted over a coaxial cable of 5mm diameter such as RG-303. This immediately limits the choice to T-44, T-50, T-68, FT-50, FT-50A, and FT-50B. We may however expect the T-44 (5.8mm hole diameter) to be a very tight fit when wound with wire of about 0.2mm diameter, and the T-68 (9.4mm hole diameter) to be a loose fit unless wound with wire of about 1mm diameter (max. 21 turns). We will therefore reject the smaller cores on the grounds that their A_L values are not significantly different from T-50 versions; and try to avoid using the larger cores unless special requirements dictate otherwise. Turning our attention to the ferrites, we may also observe that the high-permeability type-43 material has a huge temperature coefficient (1.25%/°C - which will affect low-frequency phase accuracy) and will be too lossy for medium to high-power applications (see loss factor vs frequency curves in ref [**38**]).

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[38] **Amidon Associates Inc.** (Technical data book) Jan 2000.
Technical data for iron powder and ferrite cores, including A_L values, frequency ranges, wire packing tables, Q curves, etc. Maximum flux density calculations and recommendations: p1-35. Information is also available online from: **www.amidoncorp.com** .

The low-permeability type-67 material also has no great advantage over powdered iron (it is slightly less lossy than iron, but the benefit is marginal). We will therefore consider five primary candidate cores: T50-2, T50-3, FT50-61, FT50A-61, and FT50B-61, and two secondary candidates: T68-2 and T68-3. We may now perform calculations to find the numbers of turns on each of these cores that will give at least 4.42μH (i.e., X=50Ω @ 1.8MHz); but for the sake of those who wish to design RF ammeters, or who want to maintain bridge sensitivity at LF, we will also tabulate results for various multiples of this inductance (but see also: the maximally-flat current transformer). When the A_L value is specified in nH/turns², the inductance of a winding on a particular core is given by:
$L / nH = A_L N^2$
and the required number of turns for a given inductance is:

N = √(L / A_L), where L is in nH, and A_L is in nH/turns².

Table 16.9. Turns required for target inductance. (Approximate wire lengths, computed from core dimensions, are shown in brackets below the numbers of turns).

X @ 1.8MHz → L → Core	A_L/nH	50Ω 4.42µH	100Ω 8.84µH	150Ω 13.26µH	300Ω 26.5µH	350Ω 30.9µH	700Ω 61.9µH
T50-2	4.9	30.0 (55cm)	42.5 (79cm)	52.0 (96cm)	73.6 (1.36m)	79.5 (1.47m)	112.4 (2.08m)
T68-2	5.7	27.8 (61cm)	39.4 (87cm)	48.2 (1.05m)	68.2 (1.48m)	73.7 (1.61m)	104.2 (2.27m)
T50-3	17.5	15.6 (30cm)	22.5 (43cm)	27.5 (52cm)	38.9 (72cm)	42.1 (78cm)	59.5 (1.11m)
T68-3	19.5	15.1 (33cm)	21.3 (46cm)	26.1 (57cm)	36.9 (81cm)	39.8 (87cm)	56.3 (1.22m)
FT50-61	68.8	8.1 (15cm)	11.4 (22cm)	14.0 (27cm)	19.8 (38cm)	21.3 (40cm)	30.2 (57cm)
FT50A-61	75	7.7 (18cm)	10.1 (22cm)	13.3 (28cm)	18.8 (41cm)	20.3 (43cm)	28.7 (63cm)
FT50B-61	150	5.4 (21cm)	7.7 (27cm)	9.4 (34cm)	13.3 (48cm)	14.3 (51cm)	20.3 (69cm)

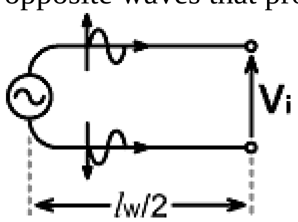
Previously it was mentioned that for operation up to 30MHz, the winding length should be kept well below 1.2m, and for operation up to 54MHz, the winding should be shorter than 67cm. It should also be said that, while transformers can be expected to give unacceptable phase errors at high frequencies if these lengths are exceeded; we cannot be sure that the phase performance will be acceptable if they are not. Some form of HF phase compensation will usually be necessary, and the fewer the turns the better it will be. We should also note, that although it is not always stated in the manufacturer's data, there is a tolerance associated with core A_L values; and so turns should be calculated for an inductance of about 20% higher than the minimum acceptable value, and the actual inductance of the winding should preferably be measured for the purpose of calculating frequency compensation networks. The origin of the '10 to 40 turns' rule-of-thumb is therefore apparent from the data in the table above. Notice also, that in the case of the FT50-series beads, increasing the bead thickness increases the wire length faster than it increases the inductance, which means that the thicker (A and B) beads have no advantage over the standard FT-50 unless power transmission is important. Since, in designing current-monitoring transformers, we are only interested in abstracting about 1 Watt from the generator, power throughput is not a major consideration (and core saturation is impossible in this application with any of the available materials). We should recall however, that core loss occurs in the absence of any secondary winding, and a substantial part of this will appear as an additional resistive component in parallel with the primary impedance. Consequently, the low permeability beads are to be preferred when monitoring the outputs of high-power transmitters. Also there may be situations in which we desire to use a transformer identical to the current transformer for voltage sampling, in which case the more stringent flux density considerations for the voltage transformer will dictate the design of the current transformer.

It is, of course, always instructive to see what others have done before embarking on a design, and so we will finish this section by reviewing some actual transformers. A current transformer described in the ARRL antenna book [19th edition 2000, ch.27, p10] uses an Amidon T50-3 (12.7mm diam. μ_i=35) powdered iron core with 31 turns of 24 AWG wire on the secondary and a secondary load of 50Ω. The A_L value of 17.5nH/turn² for this core predicts an inductance of 16.8µH for the secondary winding, and a reactance of +190Ω at 1.8MHz. Using the low-frequency analysis developed in section **10**, η=190/√(50²+190²)=0.967, corresponding to a drop in output of 3.3% or 0.29dB at 1.8MHz. The transformer is usable from 1.8-54MHz, and is stated to give an output that is flat within 0.3dB from 1.8-50MHz with the generator working into a 50Ω load. This transformer may be used with signal power levels up to 1.5KW from 3.5 to 30MHz, with reduced power handling at 1.8MHz. Increasing the core size to T68-2 (17.5mm diam, μ_i=10) and using 40 turns of 26-30 AWG wire on the secondary gives 1.5KW signal handling at 1.8MHz, but reduces the upper frequency limit to around 30MHz. The A_L value of the T68-2 core is 5.7nH/turn², corresponding to an inductance of 9.12µH for 40 Turns, and a reactance of +103Ω at 1.8MHz. Thus η=103/√(50²+103²)=0.9, giving a drop in output of 10% or 0.92dB at 1.8MHz. Clearly, both of these transformers give good performance in high-power applications, but the output voltages are somewhat low at the 100W level (1.414A primary current), being 2.28V RMS for the 31 turn version and 1.76V RMS for the 40 turn version; which leads to the need for a linearised detector [see **diode detectors**] or a fixed-gain arrangement with a non-linear meter scale in some applications. In order to increase the voltage output it is necessary to reduce the number of turns and use a higher permeability (i.e., ferrite) core, thus reducing the power handling capability. A test transformer, consisting of 10 turns wound on an FT50-61 core (12.7mm diam. μ_i=125) and loaded with 50Ω, gave a relative amplitude response that was consistent with the ideal transformer with secondary inductance model from 1.8 to at least 30 MHz, but is suitable only for power levels of around 100W (continuous) in 50Ω systems. The calculated output of this transformer however is 7.07V RMS when the primary current is 1.414A. The A_L value for the FT50-61 core is 68nH/turn², giving an inductance of 6.8μH for 10 turns, and a reactance of +77Ω at 1.8MHz. In this case $\eta=77/\sqrt{(50^2+77^2)}=0.84$, corresponding to a drop in output of 16% or 1.53dB at 1.8MHz.

13. Current-transformer propagation delay.

It is often stated in the technical literature that the self-capacitance of a single-layer coil arises from the capacitance between adjacent turns. This idea was put forward in the early part of the 20th Century; but the experimental 'evidence' in favour was shown to be fraudulent by R. G. Medhurst [ref.] in 1947. Bizarrely, the self-same theory resurfaced as new research in 1999, and got through the peer-review process despite citing the paper that refuted it. In fact, there is good reason for supposing that a coil has very little self-capacitance as such, because the properties of inductors arise from the propagation of electromagnetic energy along the winding wire. The DC conception of energy stored in a magnetic field is a special limiting case of energy stored in an EM wave, which is detained in the coil due to the finite time it takes to make its convoluted journey along the wire. The idea of capacitance between the turns then falls down because the electric vector of the travelling wave is perpendicular to the coil axis, i.e., there is very little E-field component parallel to the axis and so very little capacitance in the conventional sense. 'Self capacitance' is instead a convenient fiction, which allows us to cling to the concept of lumped inductance and still account for the fact that the coil has a self-resonant frequency (well, many in fact, but the lumped-component theory becomes inaccurate as we approach the lowest one). The first self-resonance occurs when the wire used to wind the coil is one electrical half-wavelength long, because this is the condition that allows the wave to arrive back at its starting point in phase with itself. The effective propagation velocity at this frequency is also surprisingly close to the speed of light, but the issue is complicated because the coil is a dispersive transmission-line, i.e., the velocity-factor changes with frequency. Hence, self-capacitance measurements, which depend on resonating the coil against a known capacitance and noting the difference between the actual f_0 and the f_0 predicted from the inductance, give different answers depending on the measurement frequency. Fortunately, provided that we keep well away from the open-circuit resonance frequency (which can be calculated fairly accurately just from the length of the wire), the velocity is reasonably constant, which means that the apparent self-capacitance is reasonably constant (even if it no-longer predicts the self-resonance frequency), and we can just about get away with the concepts of lumped inductance and capacitance for the purposes of circuit design.

In order to deduce an expression for the propagation delay that occurs in a transformer secondary winding, we can start by considering the effect of a magnetic disturbance half-way along the wire. This disturbance applies equal-and-opposite electromagnetic forces to the conductors leading away from it, resulting in two equal-and-opposite waves that propagate towards the transformer terminals (see diagram right).



The two waves combine at the terminals to produce a finite output voltage because they cause opposing displacements of the potentials at the terminals.

Now consider what happens when the distances travelled by the two waves are not equal (i.e., when the magnetic disturbance occurs at an arbitrary point in the coil). We can work out what happens by observing that the transformer is a linear system (neglecting core-saturation effects, which only occur under extreme conditions). This means that there is no mixing between components at different frequencies, and we can analyse the behaviour at an arbitrary frequency to deduce (within reason) what happens at all frequencies. So we may regard our two waves as sine waves of the same frequency, and the sum of two such waves is always another sine wave. If the distance to the terminal in the (arbitrarily defined) upstream direction is less than the distance in the downstream direction; then the upstream wave arriving at the terminals will be advanced relative to the downstream anti-wave; but the two waves will combine to produce an average phase that is the same as it would have been had the disturbance occurred at the exact mid point. Hence, regardless of the point of disturbance, the output of the transformer has a phase dictated by the average distance to the terminals, and this is always half the length of the winding wire (i.e., $\ell_w/2$).

In fact, transformers would not work if the output phase was not substantially the same for magnetic changes at all points on the wire; because a change in magnetisation of the core causes nearly simultaneous magnetic disturbances at all points in the coil. Hence we can consider the output to arise from an infinite number of magnetic disturbances, all of which produce their effect at the terminals in phase and therefore add together. If the arrival phases were not all the same, there would be considerable cancellation, resulting in a loss of output. As it transpires, we already have a name for this loss of coupling caused by imperfect linking between the core and the winding; it is called *leakage inductance*.

So the output of the transformer is retarded by an amount equal to the time it takes for an electromagnetic wave to travel half the length of the winding wire. In fact, this retardation occurs in both primary and secondary windings; but in a current transformer with a single-turn primary, the additional primary-side delay is relatively small. If we call the propagation time t_p, and the velocity v, noting that the units of velocity are [distance]/[time], we have:

$v = \ell_w / (2\ t_p)$

The velocity of propagation of an electromagnetic wave is:

$v = 1 / \sqrt{(\mu\ \varepsilon)}$

where μ is the permittivity and ε is the permeability of the environment. We can break this relationship down further by noting that permittivity is the product of relative and free-space permittivities, and likewise for permeability. Hence:

$v = 1 / \sqrt{(\mu_0\ \mu_r\ \varepsilon_0\ \varepsilon_r)}$

But $1/\sqrt{(\mu_0\ \varepsilon_0)}$ is the velocity of light, c. Also, the quantity $\sqrt{(\mu_r\ \varepsilon_r)}$ has a name with that some people may be familiar, it is the *refractive index*, n (and 1/n is the velocity factor). Thus we have:

$c / n = \ell_w / (2\ t_p)$

i.e.,

$t_p = n\ \ell_w / (2\ c)$ . . . . (**13.1**)

Now, consider the transformer operating at a frequency f. The time-per-cycle (period, t) is 1/f. The *time-delay* in the transformer is the negative of the propagation time, and so the delay expressed as a fraction of one cycle is:

$\Delta t / t = -t_p / t = -f\ t_p$

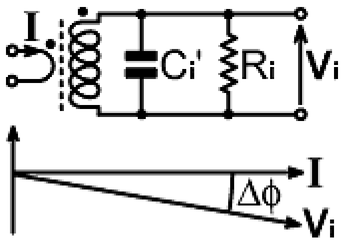
There are 2π radians in one cycle, and so the delay expressed as an angle in radians is:

$\Delta\varphi = -2\pi f\ t_p$ . . . . (**13.2**)

The phase shift is negative. The time-delay manifests itself as a capacitive effect. This capacitance is fictitious in the sense that it does not exist for static electric fields; but it is genuine in the sense that energy put into the transformer does not emerge immediately and so is temporarily stored.

Precise measurements of current-transformer HF phase error vs. frequency are given in [Evaluation and optimisation of RF current transformer bridges, section 16a]. When the effects of transformer secondary inductance and ferrite permeability dispersion are removed from the data, Δφ vs. f is a straight-line graph. This is as predicted by equation (**13.2**); i.e., the experimental data support the view that, when other contributions are controlled, the remaining phase error is due to propagation delay.

In order to represent the time delay as a parasitic capacitance, we note that the observable phase shift can be reproduced by placing a capacitance C_i' in parallel with the secondary terminals.



The phase shift depends on the load resistance and is given by:

$\Delta\varphi = \text{Arctan}(R_i / X_{Ci'})$

i.e.:

$\text{Tan}(\Delta\varphi) = -2\pi f\ C_i'\ R_i$

Hence, using (**13.2**):

$-2\pi f\ C_i'\ R_i = \text{Tan}(-2\pi f\ t_p)$

But for small angles, Tan(x)→x (when x is in radians). Hence, to a reasonable approximation:

$C_i'\ R_i = t_p$

Using equation (13.1) then gives:

$C_i' = n\,\ell_w / (2\,R_i\,c)$	(13.3)

So, we can estimate C_i' provided that we can put a number to the refractive index n (or the velocity factor 1/n). This is an intriguing problem, because it forces us to consider the medium in which the electromagnetic energy is travelling. Practically none of the energy will be inside the wire because a conductor cannot support an electric field (this being the reason for the skin effect). Hence the energy must be distributed around the outside of the conductor, and to some extent within the magnetic core material.

For a transformer core, we can estimate the refractive index as $\sqrt{(\mu_i\,\varepsilon_r)}$, where μ_i is the initial permeability. The permittivity of magnetic materials is somewhat harder to come by, but data given by Snelling [**48**] suggest that it is in the 10 to 100 range for NiZn ferrites. For the experiments described in [Eval & Opt.] type 61 ferrite, which has μ_i=125, was used. If we take ε_r=20 as a fair approximation, this gives n=50, or a velocity factor 1/n=0.02. Thus, if the energy propagating along the wire were concentrated in the core, we would expect an enormous propagation delay.

Ref:
[xx] **"RF Auto-transformers - Transmission Line Devices modelled using SPICE"**, Nic Hamilton, G4TXG, Electronics World, Nov. 2002, p52-56. Dec. 2002 p20-26.
Part 1: Limitations of the conventional transformer model. Transmission line model. Part 2: Core losses. Winding resistance.
[48] **Soft Ferrites: Properties and Applications**. E C Snelling. 2nd ed. Butterworth. 1988. ISBN 0-408-02760-6. Permittivity of ferrites: p127 - 129.

In fact, the propagation delay is modest. It is difficult to separate the phase shift due to delay from other sources of HF phase error, but from the author's data it appears that the refractive index is around 1.2 (velocity factor = 0.83). This suggests that the energy is primarily concentrated in the wire insulation and the air around the wire. It also indicates that we should use wire with very thin (and preferably non-polar) insulation. Previously, the use of plastic-coated wire was criticised on the basis that it stuffs the core with insulator and makes less room for the conductor. Now we criticise it on the basis that it increases propagation time. Use enamelled wire (or perhaps even bare wire, if you can be sure that adjacent turns won't touch).

Hence, for the purpose of estimating the secondary self-capacitance of transformers wound with enamelled wire on type 61 ferrite (and probably accurate enough for other core types as well), equation (13.3) becomes:

$C_i' = 1.2\,\ell_w / (2\,R_i\,c)$	(13.3a)

Combining the constants, using c=299729458 m/s, results in the simple formula:

C_i' /pF = 2 (ℓ_w /mm) / (R_i /Ω)	(13.3b)

Although the foregoing permits us to estimate HF phase error, it is important to be aware that there is a subtle difference between parasitic capacitance and time delay. A lumped capacitance in parallel with the secondary winding provides a reasonable basis for simulating the behaviour of the transformer; but it is nevertheless an approximation, and it is obviously a fiction because it depends on R_i. The point is that a secondary parallel capacitance moves the load impedance clockwise around a circle of constant conductance as the frequency increases, whereas a time-delay moves the output phase around a circle centred on the graph origin (0,0). In other words, a pure time-delay affects only the output phase, without affecting the magnitude, whereas a parallel capacitance affects both.

It will transpire that the simple relationship between delay and self-capacitance derived above is sufficient for the purpose of building accurate bridges. Against the risk of being accused of lack of scientific rigour however, it must be said that a transformer winding is a transmission line, and a pure time delay with no attendant impedance transformation only occurs when the line is matched. We can always estimate the phase-shift from a knowledge of the line-length and the velocity-factor, but to quantify the impedance transformation it is necessary to know the characteristic resistance.

In the approximation that the line is lossless, the characteristic (surge) resistance is:
$R_{surge} = \sqrt{(L / C)}$
Where L is the inductance per unit length and C is the capacitance per unit length. It is tempting to think that the inductance of the line will be the same as the inductance of the transformer winding, but this is not the case.

Thinking back to the idea of a wave and an anti-wave created by a magnetic disturbance; note that the upstream line and the downstream line are in a state of close magnetic coupling, via the core, and that the mutual inductance is negative. This means that the inductance of the line overall is largely cancelled, just as it is when the two conductors of an ordinary transmission line are brought into proximity. Indeed, it is this ability to cancel its own inductance when energy is abstracted from the secondary that allows a transformer to work. In an ideal transformer, the cancellation is perfect, and so we may deduce that the distributed inductance of the transmission line is the *leakage inductance* of the winding.

Author's note:
Became embroiled in difficult theory at this point.

>>> writing in progress.

>>>>> ?? Speculative [no, not any more].
The leakage inductance of a winding on small toroidal transformer is usually about 1% of the total (it depends on the permeability of the core). Hence for a winding with an inductance of 10μH, we expect the leakage inductance to be of the order of 100nH. If we wish to determine the surge resistance, of course, we also need to determine the distributed capacitance; and this is another perplexing problem. The capacitance will certainly be greater than the 8.85pF/m of free space, and it will depend on the proximity of other objects (such as the Faraday shield). The best we can say (via a somewhat recursive argument) is that it will be of the order of the 'self-capacitance' C_i'. In one of the author's experimental current-transformers, with a secondary inductance of 9μH and a leakage inductance of about 90nH, it was estimated that C_i' was about 9pF. Hence, for this transformer, a fair guess for $\sqrt{(L/C)}$ is that it is of the order of 100Ω. This astonishing result suggests that the empirically-derived wisdom that small RF transformers give nearly ideal performance when terminated in 50Ω (or thereabouts) is influenced by the underlying transmission-line behaviour. It means that the impedance-transformation occurring within the winding, seen as a deviation from ideality, is small or negligible. The line is very-nearly matched when the load is a few tens of Ohms; and the HF phase-shift, having minimal associated change in output magnitude, looks like an almost-pure time-delay.

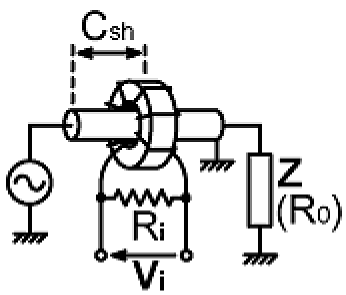
>>>>>
Is this true? The measurements of [Eval & Opt. - sec.16a] suggest that it is. Sheath helix (slow-wave) theory, on the other hand suggests that the time delay should, in principle, vary with frequency. Further investigation is required, but it appears that the superposition of the axial slow-wave and the superluminal helical wave results in a wave that does have a phase velocity close to c.

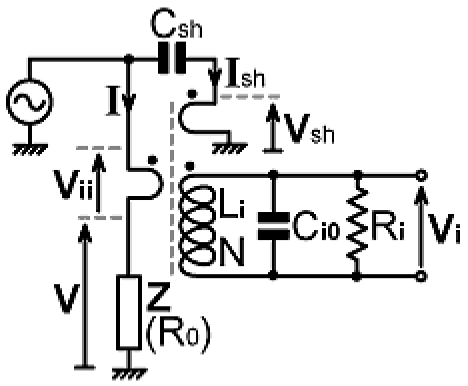
>>>
Gap capacitance, lead-wire capacitance and pitch-angle effect in coils of low N.

14. Effective secondary capacitance:
Although propagation delay makes a substantial contribution to the effective secondary capacitance required for modelling purposes; it is by no means the whole of the story. There are various other effects, some of which increase the apparent capacitance, and some of which reduce it and can make it negative overall. Failure to recognise and quantify these influences gives rise to inconsistencies of performance

>>>>>
The topics for the next few sections are to be based on the experimental work described in [Eval & Opt.].

14a. Faraday-shield protrusion capacitance.





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Through-line mismatch.
Secondary load reactance.
Perturbation series for C_i.

15. HF Neutralisation:

circuits that fake a transformer with no self-capacitance.

Load port compensation capacitor.
Herzog's compensation.

Quadrature current injection. Neugebauer and Perrault
Parasitic Capacitance Cancellation in Filter Inductors. T C Neugebauer and D J Perreault. 35th Annual IEEE Power Electronics Specialists Congerence, 2004. The parasitic capacitance of a power-supply filter inductor can be cancelled by use of an auxiliary winding and a capacitor.

Quadrature voltage addition
Phase-shift compensation.
Delaying the voltage sample.

u